Supplementary Information for "Frequency sidebands generation of quantum dot single photons with preserved indistinguishability"

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Phase modulators use the linear electro-optic effect to modulate the index of the material. When a monochromatic electric field with amplitude E_0 and frequency ω_0 is sent through a phase modulator, the input and output fields can be written as

$$E_{in} = E_0 e^{i\omega_0 t} \qquad \text{and} \qquad E_{out} = E_0 e^{i\omega_0 t - i\phi} \tag{1}$$

where $\phi = \frac{2\pi}{\lambda} n_x L$ is the phase factor gained by the field of wavelength λ traveling through a medium with index n_x and length L. When the modulator is driven with a sinusoidal voltage $V = V_m \sin(\Omega t + \theta)$, the index of refraction is modified due to the applied electric field [1],

$$\phi = \frac{2\pi}{\lambda}(n_x + \Delta n_x)L \tag{2}$$

For a transverse modulator, where the microwave field is applied transverse to the propagation direction of the optical field, the change in the phase factor becomes [1]

$$\Delta n_x = -\frac{1}{2} n_x^3 r \left(\frac{L}{d}\right) V_m \sin(\Omega t + \theta), \tag{3}$$

where r is the interaction coefficient and d is the thickness of the crystal across which the voltage is applied. With the modified phase, the output field can be rewritten as

$$E_{out} = E_0 e^{i\omega_0 t - i\phi_0 - i\beta \sin \Omega t} \tag{4}$$

where $\phi_0 = \frac{2\pi}{\lambda} n_x L$ is the time-independent phase factor and $\beta = \frac{\pi}{2} n_x^3 r (\frac{L}{d}) V_m$ is the modulation index, proportional to the applied voltage. The constant phase factor can be absorbed into the electric field amplitude and the exponential can be expanded as a sum of the coefficients of the Bessel functions of the first kind (J_n),

$$E_{out} = E_0 \sum_{n=-\infty}^{\infty} J_n(\beta) e^{in(\theta - \pi/2)} e^{i(\omega_0 + n\Omega)t}. \tag{5}$$

For notational simplicity, the input field can be represented with ket-notation as $|1_{\omega_0}\rangle$, such that the output state can be written as,

$$E_{out} = \sum_{n=-\infty}^{\infty} J_n(\beta) e^{in(\theta - \pi/2)} |1_{\omega_0 + n\Omega}\rangle. \tag{6}$$

A frequency qubit can be formed by suppressing the unwanted frequency components using a single sideband generation technique, where the carrier and a single sideband is generated through the modulation process [2],

$$|\psi\rangle = c_0 |1_{\omega_0}\rangle + c_1 e^{i\theta} |1_{\omega_0 + n\Omega}\rangle. \tag{7}$$

where the coefficients are normalized Bessel coefficients,

$$c_0 = \frac{J_0(\beta)}{\sqrt{J_0^2(\beta) + J_1^2(\beta)}} \quad \text{and} \quad c_1 = \frac{J_1(\beta)}{\sqrt{J_0^2(\beta) + J_1^2(\beta)}} e^{i\pi/2} \tag{8}$$

The Bessel coefficients are a function of the applied microwave voltage. By changing the voltage and the phase of the microwave field, one can rotate the qubit to an arbitrary point on the Bloch sphere. Similarly, the first two sidebands and the carrier can be used as a qutrit for implementing frequency coded BB84 protocols.

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[1] T. A. Maldonado, *Handbook of Optics*, 2nd ed. (McGRAW-HILL, INC, 2001) Chap. 13, pp. 13.1–13.33.
[2] S. Shimotsu, S. Oikawa, T. Saitou, N. Mitsugi, K. Kubodera, T. Kawanishi, and M. Izutsu, IEEE Photonics Technology Letters **13**, 364366 (2001).